

Towards Optimal Patrol Strategies for Fare Inspection in Transit Systems

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Abstract

In some urban transit systems, passengers are legally required to purchase tickets before entering but are not physically forced to do so. Instead, patrol units move about through the transit system, inspecting tickets of passengers, who face fines for fare evasion. This setting yields the problem of computing optimal patrol strategies satisfying certain temporal and spacial constraints, to deter fare evasion and hence maximize revenue. In this paper we propose an initial model of this problem as a leader-follower Stackelberg game. We then formulate an LP relaxation of this problem and present initial experimental results using real-world ridership data from the Los Angeles Metro Rail system.

Introduction

In some urban transit systems, including the Los Angeles Metro Rail system, passengers are legally required to buy tickets before boarding although there are no gates or turnstiles physically denying access to the ticketless. (There are, quite literally, no barriers to entry, as shown in Figure 1.) Instead, security personnel are deployed throughout the transit system, randomly inspecting passenger tickets; fare-evaders face significant penalties when caught.



Figure 1: Entrance of a LA Metro Rail station.

With approximately 80,000 riders daily on the LA Metro, fare evasion can cause significant revenue loss. The Los Angeles Sheriff's Department (LASD) deploys uniformed pa-

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trols on board trains and at stations for fare-checking and crime-prevention. With limited resources, however, it is impossible to cover all locations at all times, thus requiring some mechanism for choosing the times and locations for inspections to occur. Given the nature of the patrol setting, any predictable patterns in such a patrol schedule may well be observed and exploited by the potential fare-evaders. This places us in the leader-follower Stackelberg setting, in which LASD (the leader) commits to a randomized patrol strategy and potential fare-evaders (the followers) observe the patrol strategy before choosing their best courses of action. Thus we face the problem of developing patrol strategies that efficiently utilize limited resources to best deter rational, adaptive fare-evaders. That is, we seek patrol strategies that will best incentivize riders to purchase their tickets rather than evade the fare.

In this work we initiate the study of optimal deterrence of fare-evasion in urban transit systems, in settings characterized by the following assumptions:

- *Known passenger types*: The total number of passengers using the metro system as well as the distribution of passenger types is fixed and independent of the inspection strategy. Passenger type is specified by the passenger's preferred route and time of trip.
- *Fixed route*: Each passenger takes his/her preferred route at his/her preferred time regardless of the patrolling strategy.
- *Rational, risk-neutral agents*: Riders make a binary decision of buying or not buying the ticket (the decision to ride having already been made), in order to minimize their expected total cost, following for simplicity the classic economic analysis of rational crime (Becker 1968; Becker and Landes 1974). Of course, it may be that some conscientiousness riders purchase tickets irrespective of expected costs; our focus here is on providing incentives to the portion of the ridership that does weigh these choices. Although in general riders may be risk-averse (or even risk-seeking, i.e., deciding not to buy a ticket so as to gamble for fun), here we assume all riders are risk-neutral.
- *Stackelberg game*: The rider observes the patrol strategy before making the ticket purchase decision.
- *Zero-sum game*: Unlike other sorts of punishment deployed in criminal justice, fines issued result in revenue to

the state. Therefore the optimization objective we choose for the leader is to maximize total revenue to the transit system (total ticket sales plus penalties), i.e., to recover as great a fraction of the ticket price per passenger as possible.

The equilibrium strategy of the police in this zero-sum game is essentially a maximin strategy, where fare-evaders maximize their utilities as if the inspection strategy is known.

The leader-follower Stackelberg game model has been the topic of much recent research (Tambe 2011) and has been applied to a number of real-world security domains, including the Los Angeles International Airport (Paruchuri et al. 2008), the Federal Air Marshals Service (Tsai et al. 2009), and the Transportation Security Administration (Pita et al. 2011). Urban transit systems, however, present unique computational challenges. There are exponentially many possible patrol strategies, each subject to both the spatial and temporal constraints of travel within the transit network under consideration. Explicitly representing a randomized strategy (i.e. a probability distribution over all pure strategies) would be impractical.

Instead, we introduce the *transition graph*, which captures the spatial as well as temporal structure of the domain, and we solve for the optimal (fractional) flow through this graph, by linear programming (LP). Although such a flow (which we refer to as a *relaxed solution*) does not directly yield a patrol strategy, it can be interpreted as a marginal distribution from which to generate actual patrols; how to do so is a topic of our ongoing research. Moreover, because the LP formulation is a relaxation of the problem, its optimal solution value upper-bounds the true optimal, which can be used in evaluating the performance of particular strategies.

Finally, we perform simulations in which we solve for such optimal relaxed solutions in problem instances constructed based on actual ridership data provided by the LASD and publicly available timetables, for three LA Metro train lines (Blue, Gold, and Green). A distribution on rider types is estimated by sampling from the ridership level data. Our initial results suggest the possibility of significant fare evasion deterrence and hence prevention of revenue loss. We conclude by discussing plans for future work.

Setting

Train System

We now describe the problem setting in detail. In this game, a pure leader strategy is a *patrol*, i.e., a sequence of *patrol actions* (defined below), of bounded duration. The two possible pure follower strategies are buying and not buying. We formulate this problem as a Stackelberg game, with one leader and multiple followers.¹ Each follower observes the strategy the leader commits to and plays a best response. There are many follower types, one for each source, destination, and departure time triple (corresponding to the set of

¹We use interchangeably the terms *patrol strategy* and *leader strategy*, as well as the terms *rider* and *passenger*. We sometimes use the term *police* to refer to those carrying out the patrol.

all riders who take such a trip). In general the leader’s strategies will be mixed; without loss of generality the followers’ strategies can be assumed to be pure.

The train system consists of a single line (i.e., a path graph with stations as nodes) on which trains travel back and forth, in general with multiple trains traveling simultaneously. The system operates according to a fixed daily schedule, with trains arriving at stations at (finitely many) designated times throughout the day. We refer to the entire path that a given train takes through this graph, from the start station to the terminal station, as a *train path*. Therefore we can model time as slotted, focusing only on the time steps at which some (instantaneous) train arrival/departure event occurs. We use the (directed) *transition graph* $G = \langle V, E \rangle$ to encode the daily timetable of the metro line, where a vertex $v = \langle s, t \rangle$ corresponds to some pair of station s and discrete time point t .

Patrols

There are a fixed number γ of deployable patrol units, each of which may be scheduled on a patrol of duration at most κ hours (with, e.g., $\kappa = 7$). There are two sorts of patrol actions, which a given patrol unit can alternate between on its shift: *on-train* inspections (in which police ride the train, inspecting their fellow passengers), and *in-station* inspections (in which they inspect passengers as they exit the station). A pure patrol strategy is represented mathematically as a path in G . An edge e represents an *atomic* patrol action, i.e., inspecting in-station from the time of one train event at that station to the next (at that station) or inspecting on-train as it travels from one station to the next. In particular, there is an edge from $v = \langle s, t \rangle$ to $v' = \langle s', t' \rangle$ if:

- s' is either the predecessor or successor of s in the station sequence and $\langle s, t \rangle$ and $\langle s', t' \rangle$ are two consecutive stops for some train in the train schedule, or
- $s' = s$, $t < t'$, and there is no vertex $\langle s, t^\dagger \rangle$ such that $t < t^\dagger < t'$ (representing a minimal “stay at a station” action).

Each edge e has a length l_e equal to the corresponding action duration and an effectiveness value f_e , which represents the percentage of the relevant ridership inspected by this action. (For on-train inspections, this depends on the ridership volume at that location and time of day and on the duration; for in-station inspections, we assume that all exiting passengers are inspected, and so set the effectiveness to 1.) A valid pure patrol strategy is then a set of paths P_1, \dots, P_γ , each of size at most κ , i.e., $\sum_{e \in P_i} l_e \leq \kappa$.

Example 1 A simple scenario with 3 stations (A, B, C) and 4 discrete time points (7am, 8am, 9am, 10am) is given in Figure 2. The dashed lines represent staying actions; the solid lines represent traveling actions. There are 4 trains in the system; all edge durations are 1 hour. A sample train path here is $\langle A, 7am \rangle \rightarrow \langle B, 8am \rangle \rightarrow \langle C, 9am \rangle$. In this simple example, if $\kappa = 3$ and $\gamma = 1$, then the valid pure leader strategies consist of all paths of length 3.

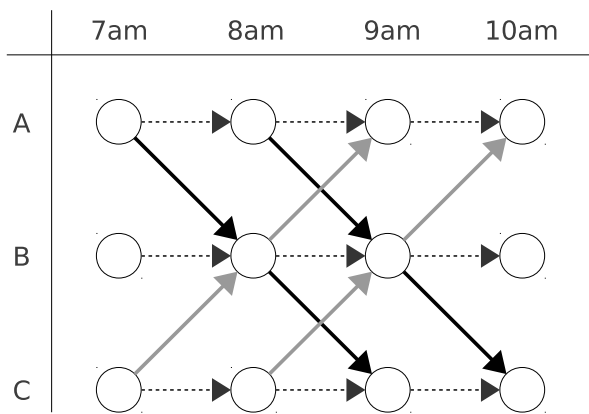


Figure 2: The transition graph of a tiny toy example problem instance.

Riders

The ticket price (for any trip within the transit system) is a nominal fee, with the fine for fare-evasion much greater. The ticket fee is fixed (if paid) but the receipt of a fine for fare-evasion is uncertain; based on the likelihood of being caught, the rider must make a decision as to whether to buy a ticket. We assume the riders are rational, risk-neutral economic actors (Becker 1968; Becker and Landes 1974), who make this choice in order to minimize expected cost. (Equivalently, we can assume that some riders are conscientious, but that selfish or rational riders are distributed evenly among all passenger types.)

A rider’s *type* is defined by the path he takes in the graph. Because there is a single train line, we assume that riders never pause in stations, i.e., do not follow any “stay” edges in the middle of the trip; on the other hand the last edge of any evader type should always be staying at the destination station for the minimum amount of time, representing the action of “exiting” the station (the rider would be inspected by the police doing in-station inspection at the same station at the same time). Therefore the space Λ of rider types corresponds to the set of all subpaths of train paths. (When G is drawn as in Figure 2, all rider paths are “diagonal” except for the last edge.) We identify a rider type λ with the corresponding path. A metro line with N stops and M trains will have a total of less than $\frac{MN(N-1)}{2}$ rider types.

Given a patrol path P , the inspection probability for a rider of type $\lambda \in \Lambda$ is:

$$\min\{1, \sum_{e \in P \cap \lambda} f_e\} \quad (1)$$

We justify this probability as follows. First, consider on-train inspections. We assume that during an on-train inspection riders are inspected in sequence, from one end of the train to the other. The fraction of the train that is inspected in a given inspection action will depend on the ridership volume and the duration of the segment. Given sufficiently many consecutive on-train inspection actions, therefore, the patrol will work its way through the entire train and capture all (remaining) fare-evaders. Therefore, unlike in the setting

where inspections choose a random sample of train riders, the probabilities are added rather than multiplied. Now also consider in-station inspections. Since a rider of course only leaves one station when taking a single trip, a rider will encounter at most one in-station inspection. We assume that in an in-station inspection, *all* departing riders are inspected, and so (1) remains valid.²

Objective

The leader’s utility, equal to total expected revenue, can be decomposed into utilities from bilateral interactions against each individual follower. This implies that the game is payoff-equivalent to a Bayesian Stackelberg game between one leader with one type and one follower with multiple possible types. Specifically, we denote the prior probability of a rider type $\lambda \in \Lambda$ (proportional to its ridership volume) by p_λ .

Furthermore, these utility functions imply that the game is zero-sum, in which case the Stackelberg equilibrium is equivalent to the max-min solution. Although such zero-sum Bayesian games are known to be solvable by linear programming (Ponssard and Sorin 1980), that approach would be impractical here due to the exponential number of pure leader strategies.

LP Formulation

In this section, we formulate a linear program which defines a relaxation of the problem of choosing a maximum-revenue patrol strategy. As noted above, the leader’s space of pure strategies is exponentially large. Therefore we instead compactly represent patrol strategies by marginal probabilities on edges x_e of the transition graph, i.e., by the probabilities that inspection actions appear within the chosen patrol path.

In order to be able to restrict our attention to unique source and sink nodes, we add to the transition graph a source v^+ with edges to all possible vertices where a patrol can start and a sink v^- with edges from all possible vertices where a patrol can end. (These additional dummy edges have zero duration and zero effectiveness.) We denote the set of possible starting vertices reachable from v^+ by V^+ and the set of possible ending vertices leading to v^- by V^- . We also use notation V^t to denote the set of vertices whose discrete time component is less than or equal to t .

Then the following linear program provides an upper

²In a more general setting in which either in-station inspections inspect only a random fraction of exiting passengers or if on-train inspections are performed randomly (with replacement), (1) will upper-bound the true inspection probability.

bound on the optimal revenue achievable:

$$\begin{aligned} \max_{\mathbf{x}, \mathbf{u}} \quad & \sum_{\lambda \in \Lambda} p_{\lambda} u_{\lambda} \\ \text{s.t.} \quad & u_{\lambda} \leq \min\{\rho, \tau \sum_{e \in \lambda} x_e f_e\}, \text{ for all } \lambda \end{aligned} \quad (2)$$

$$\sum_{e \in E} l_e \cdot x_e \leq \gamma \cdot \kappa \quad (3)$$

$$\sum_{v \in V^+ \cap V^t} x_{(v^+, v)} \leq \sum_{v' \in V^- \cap V^{t+\kappa}} x_{(v', v^-)}, \text{ for all } t \quad (4)$$

$$\sum_{v \in V^+} x_{(v^+, v)} = \sum_{v \in V^-} x_{(v, v^-)} \leq \gamma \quad (5)$$

$$\sum_{(v', v) \in E} x_{(v', v)} = \sum_{(v, v^{\dagger}) \in E} x_{(v, v^{\dagger})}, \text{ for all } v \quad (6)$$

$$0 \leq x_e \leq 1, u_{\lambda} \geq 0 \quad (7)$$

Here u_{λ} denotes the expected value paid by a rider of type λ , and so $v_{\lambda} u_{\lambda}$ is the expected total revenue from riders of this type; x_e is the probability of a patrol on edge e .

Constraint (2) indicates that the rider will best-respond, by bounding the expected cost to a rider of type λ by both the ticket price and the expected fine if such a rider chooses not to buy. Here $\sum_{e \in \lambda} x_e f_e$ equals the expected number of times that a rider of this type will be inspected, which upper-bounds the probability of inspection, hence indeed upper-bounding the amount that this rider will pay. (The 1 from (1) can be omitted because $\rho < \tau$.)

Constraint (3) limits the total number of time units to $\gamma \cdot \kappa$; moreover, the total flow that enters the system before time t must exit the system before time $t + \kappa$, as restricted by Constraint (4). (Both these constraints are less strict than limiting each of the γ units to duration κ .) Constraint (5) indicates that the total flow allowed to enter the system and the total flow that must exit the system must be equal *and* less than or equal to γ , the number of total units allowed. Finally, Constraint (6), enforces conservation of flow, which clearly is satisfied by any mixed patrol strategy.

Evaluation

This section presents our initial evaluation based on real metro schedules and rider traffic data. The linear program given above is solved using CPLEX 12.2 on a standard 2.8GHz machine with 4GB main memory. We first describe the data sets used, followed by experimental results.

Data Sets

We created three different data sets, each using a Los Angeles Metro Rail line: Blue, Gold, and Green. For each line, we created its temporal graph using the corresponding timetable available at <http://www.metro.net>. Implementing the LP requires a fine-grain ridership distribution of potential fare evaders (recall that in our LP formulation, a follower type corresponds to a pair of boarding station / time and disembarking station / time). Because the distribution of fare evaders among the passenger population is difficult to

obtain since even when a fare evader is caught, his or her intended trip may not be revealed, we assume (as stated above) that potential fare-evaders are evenly distributed among the general population, for which ridership statistics are made available to us by LASD.

In our experiments, we create the follower distribution using hourly boarding and alighting counts derived from data provided by the Los Angeles Sheriff Department. The LASD data includes boarding and alighting counts for every hour; we sample based on these to create a fine-grain ridership distribution. The inspection effectiveness f_e of an edge is set based on the assumption that 10% of a train can be inspected per minute,³ capped at 1, i.e., $f_e = \min\{.1 \cdot l_e, 1\}$. The ticket fare is set to \$1.5 (the actual current value) while the fine is set to \$100.⁴ Table 1, 2, and 3 summarize the detailed statistics for each individual Metro line.

# of stations	# of trains	Avg. # of daily passengers
20	287	76631
# of vertices	# of edges	# of follower types
4986	9890	44632

Table 1: Metro Blue Line Statistics.

# of stations	# of trains	Avg. # of daily passengers
13	257	22148
# of vertices	# of edges	# of follower types
3227	6298	19583

Table 2: Metro Gold Line Statistics (prior to the opening of the east Los Angeles extension).

# of stations	# of trains	Avg. # of daily passengers
14	217	38314
# of vertices	# of edges	# of follower types
2891	5681	19045

Table 3: Metro Green Line Statistics.

Experimental Results

In our experiments, we fix the number of inspectors to 1 but vary the maximum number of hours that an inspector can patrol from 4 hours to 7. Figure 3(a) shows the total runtime required to set up the LP, including the creation of the temporal graph and the conversion to the LP. As can be seen, it takes significantly longer to create the LP for the Blue line than for the other two lines since the Blue line is considerably larger than the others in terms of graph size and (thus)

³In these initial experiments, the ridership volume is for simplicity ignored here.

⁴In fact, fare evaders in Los Angeles can be fined \$200, but they also may be issued warnings. Of course, if we could increase the fine dramatically the rider would have much less incentive for fare evasion, and we can achieve better revenue. However a larger fine might not be feasible legally.

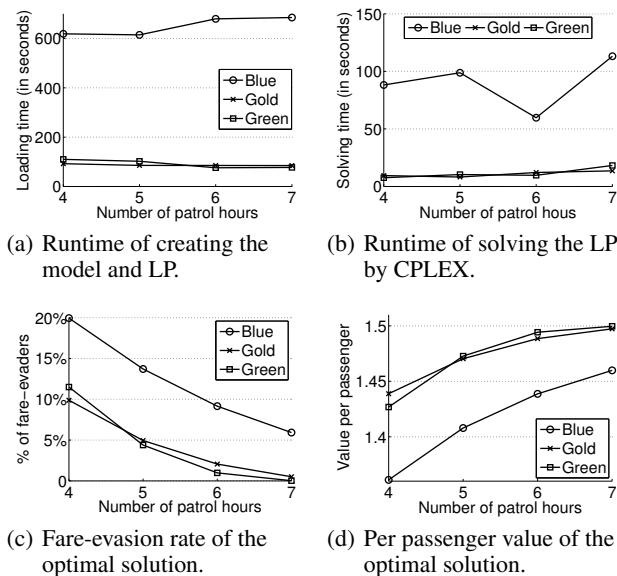


Figure 3: Experimental results.

number of follower types. Figure 3(b) shows the runtime required by CPLEX to solve the LP created. Again, solving the LP for the Blue line takes significantly longer than solving for the other two lines since it has more variables and constraints due to a larger graph size. Increasing the number of patrol hours seems to make the problem harder—the runtime increases almost consistently for all the three lines (with the only exception at the Blue line with six patrol hours).

Figure 3(c) shows the percentage of fare-evaders in the optimal (*relaxed*) solution. Recall that in our game-theoretical formulation, a passenger will fare-evade if and only if the expected fine is lower than the ticket price. Given the fine / fare ratio of 100 : 1.5, all passengers inspected with probability lower than 1.5% will become fare-evaders, while the rest will always buy tickets. As can be seen in Figure 3(c), the fare-evasion rate decreases almost linearly in the number of additional patrol hours beyond 4. An optimal 7-hour patrol (*relaxed*) strategy can lower the fare-evasion rate to 6% for the Blue line, 0.5% for the Gold line, and 0.03% for the Green line.

Finally, Figure 3(d) shows the expected value per passenger in the optimal (*relaxed*) solution. The passenger can always pay the ticket price for \$1.5 and will only evade the ticket when the expected fine is lower. Hence the theoretical maximum achievable is \$1.5 when every passenger buys the ticket. As we can see, the per passenger value increases with number of patrol hours, converging to the theoretical upper bound of \$1.5. An optimal 4-hour (*relaxed*) patrol strategy can already provide reasonably good expected value: 1.36 for the Blue line (90.7% of the maximum), 1.44 for the Gold line (95.9%), and 1.43 for the Green line (95.1%).

Conclusions and Future Research

In this paper we presented initial results from a research effort on generating fare-inspection strategies in urban transit systems. We modeled the domain as a Stackelberg game,

and by compactly representing the leader’s mixed strategies as flows in the transition graph, we were able to formulate a relaxation of the problem as a compact LP. We find in our simulations that our computed (*relaxed*) solutions effectively deter fare evasion and ensure high levels of revenue.

We plan to continue developing and eventually deploy our system with the LA Metro system. Towards this goal, many interesting and important research questions remain open. We list some of them below.

- **Sampling from marginal probability distributions to obtain feasible patrols.** The solutions of our LP are marginals, i.e., expected coverage of each edge in the transition graph. The next task is to compute the actual randomized strategy, either as a mixture of a small number of patrols, or as a sampling procedure that outputs a patrol drawn from some distribution, ideally such that the resulting expected coverage of the edges matches the LP solution. This is a topic of our ongoing research.
- **Relaxing the zero-sum assumption.** In our initial model described here, we assumed that utilities correspond to the total monetary transfer between passengers and transit authority, resulting in a zero-sum game. It would be interesting to explore non-zero-sum models, in order to model non-risk-neutral fare evaders and/or leaders with additional objectives, such as minimizing crime.
- **Exploring other strategic options.** So far we have modeled each passenger’s choice as either buying or not buying the ticket. In practice they could have other options, such as taking an earlier or later train, boarding and/or disembarking at different (but nearby) stations, or deciding against riding the metro altogether. The opposite could also happen where some currently non-riding person decides to become a rider.
- **Modeling human behavior.** The passengers are human decision-makers and thus might not be perfect optimizers. Incorporating such behavior models of the adversary in security games (Pita et al. 2010; Yang et al. 2011) may potentially increase the robustness of our solutions.
- **Beyond static models.** In practice, this game is (of course) repeated over time. If we can get sufficiently informative data on the total revenue, this would allow us to evaluate different leader models of fare-evaders, corresponding to different leader strategies, and choose the best one. One potential direction would be to automate this process of picking the best model of the fare-evaders based on the historical performance of the models, utilizing ideas from online/reinforcement learning and opponent modeling.

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