Team Formation in Large Action Spaces

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Abstract. Recent work has shown that diverse teams can outperform a uniform team made of copies of the best agent. However, there are fundamental questions that were not asked before. When should we use diverse or uniform teams? How does the performance change as the action space or the teams get larger? Hence, we present a new model of diversity for teams, that is more general than previous models. We prove that the performance of a diverse team improves as the size of the action space gets larger. Concerning the size of the diverse team, we show that the performance converges exponentially fast to the optimal one as we increase the number of agents. We present synthetic experiments that allow us to gain further insights: even though a diverse team outperforms a uniform team when the size of the action space increases, the uniform team will eventually again play better than the diverse team for a large enough action space. We verify our predictions in a system of Go playing agents, where we show a diverse team that improves in performance as the board size increases, and eventually overcomes a uniform team.

Keywords: Coordination & Collaboration, Distributed AI, Team Formation

1 Introduction

Team formation is crucial when deploying a multi-agent system [1–4]. Many researchers emphasize the importance of diversity when forming teams [5–8]. However, there are many important questions about diversity that were not asked before, and are not explored in such models.

LiCalzi and Surucu (2012) [5] and Hong and Page (2004) [7] propose models where the agents know the utility of the solutions, and the team converges to the best solution found by one of its members. Clearly in complex problems the utility of solutions would not be available, and agents would have to resort to other methods, such as voting, to take a common decision. Lamberson and Page (2012) [6] study diversity in the context of forecasts, where the solutions are represented by real numbers and the team takes the average of the opinion of its members. Domains where the possible solutions are discrete, however, are not captured by such a model.

Marcolino, Jiang, and Tambe (2013) [8] study teams of agents that vote in discrete solution spaces. They show that a diverse team of weaker agents can overcome a uniform team made of copies of the best agent. However, this phenomenon does not always occur, and they do not present ways to know when we should use diverse teams. Moreover, they lack a formal study of how the performance of diverse teams change as the number of agents and/or actions increase.

In this paper we shed new light on this problem, by presenting a new, more general model of diversity for teams of voting agents. Our model captures better than the previous ones the notion of a diverse team as a team of agents that tend not to agree on the same actions, and allows us to make new predictions. Our main insight is based on the notion of *spreading tail* (ST) and *non-spreading tail* (NST) agents. As we will show, a team of ST agents has a diverse behavior, i.e., they tend to not agree in the same actions. Hence, we can model a diverse team as a team of ST agents, and show that the performance improves as the size of the action space gets larger. We also prove upper and lower bounds on how fast different teams converge. The improvement can be large enough to overcome a uniform team of NST agents, even if individually the ST agents are weaker. As it is generally hard to find good solutions for problems with a large number of actions, it is important to know which teams to use in order to tackle such problems. Moreover, we show that the performance of a diverse team converges to the optimal one exponentially fast as the size of the team grows. We show synthetic experiments that provide even further insights about our model. Even though the diverse team overcomes the uniform team in a large action space, the uniform team eventually will again play better than the diverse team as the action space keeps increasing if the best agent does not behave exactly like an NST agent.

Finally, we test our predictions by studying a system of voting agents, in the Computer Go domain. We show that a uniform team made of copies of the best agent plays better in smaller board sizes, but is overcome by a diverse team as the board gets larger. Moreover, we analyze the agents and verify that weak agents have a behavior closer to ST agents, while the best agent is closer to an NST agent. Therefore, we show that the predictions are verified in a real system, and can effectively be used while forming a multi-agent team.

2 Related Work

This paper is mainly related to team formation. However, we can also find related work in the study of voting and ensemble systems. We will start by focusing on team formation research. There are many models about the importance of diversity when forming teams, both in the economics and in the multi-agent systems literature [5–8]. These were already discussed in the previous section, so we will not discuss them again here.

In classical team formation research, the team with maximum expected utility is chosen, based on a model of the capabilities of each agent [1, 2]. However, in many domains we do not have such a model. The study of "ad-hoc" teamwork deals with multi-agent teams with absence of information [9, 10]. They focus, however, on how a new agent must decide its behavior in order to cooperate with agents of unknown type, not on picking the best team.

Concerning voting, this paper is related to the view of voting as a way to discover an optimal choice (or ranking). Classical models study this view of voting only for teams of identical agents [11, 12]. However, more recent works are also considering agents with different probability distribution functions (pdfs). Caragiannis, Procaccia, and Shah (2013) [13] study which voting rules converge to a true ranking as the number of agents (not necessarily identical) goes to infinity. In [14] the problem of inferring the true ranking is studied, assuming agents with different pdfs, but drawn from the same family. Even though recent works on voting are not assuming identical agents, they still do not provide a way to find the best teams of voting agents.

More related works can be found in the study of ensemble systems. These are the construction of a strong classifier by combining multiple weak classifiers, for example by voting [15]. An important problem is how to form the ensemble system, i.e., how to pick the classifiers that lead to the best predictions [16]. Our model, based on the notion of *spreading tail* and *non-spreading tail* agents allow us to make many predictions about teams as the action space and/or number of agents change, and also compare the rate of change of the performance of different teams. To the best of our knowledge, there is no model similar to ours in the ensemble system literature.

3 Model for Analysis of Diversity in Teams

Consider a problem defined by choosing an action a from a set of possible actions $\mathbf{A}$. Each a has an utility $U(a)$, and our goal is to maximize the utility. We always list the actions in order from best to worst, therefore $U(a_j) > U(a_{j+1})\ \forall j$ (a_0 is the best action). In some tasks (like in Section 4), a series of actions are chosen across different states, but here we focus on the decision process in a given state.

Consider a set of agents, voting to decide over actions. The agents do not know the utility of the actions, and vote for the action they believe to be the best according to their own decision procedure, characterized by a probability distribution (pdf). We write as $p_{i,j}$ the probability of agent i voting for action a_j. We denote by $p_{i,j}(m)$, when we explicitly talk about $p_{i,j}$ for an action space of size m. If the pdf of one agent is identical to the pdf of another agent, they will be referred to as copies of the same agent. The action that wins by plurality voting is taken by the team. Ties are broken randomly, except when we explicitly talk about a tie breaking rule. Let $\mathbf{D_m}$ be the set of suboptimal actions ($a_j, j \neq 0$) assigned with a nonzero probability in the pdf of an agent i, and $d_m = |\mathbf{D_m}|$. We assume that there is a bound in the ratio of the suboptimal action with highest probability and the one with lowest nonzero probability, i.e., let $p_{i,min} = min_{j \in \mathbf{D_m}} p_{i,j}$ and $p_{i,max} = max_{j \in \mathbf{D_m}} p_{i,j}$; there is a constant α such that $p_{i,max} \leq \alpha p_{i,min}\ \forall$ agents i.

We define strength as the expected utility of an agent and/or a team. The probability of a team playing the best action will be called p_{best}. We first consider a setting where $U(a_0) \gg U(a_j) \forall j \neq 0$, hence we can use p_{best} as our measure of performance. We will later consider more general settings, where the first r actions have a high utility.

We define team formation as selecting from the space of all agents a limited number of agents that has the maximum strength by voting together to decide on actions. We study the effect of increasing the size m of the set of possible actions on the team formation problem. Intuitively, the change in team performance as m increases will be affected by how the pdf of the individual agents i change when m gets higher. As we increase m, d_m can increase or not change. Hence, we classify the agents as *spreading tail* (ST) agents or *non-spreading tail* agents (NST).

We define ST agents as agents whose d_m is non-decreasing on m and $d_m \to \infty$ as $m \to \infty$. We consider that there is a constant $\epsilon > 0$, such that for all ST agents i, $\forall m$, $p_{i,0} \geq \epsilon$. We assume that $p_{i,0}$ does not change with m, although later we discuss what happens when $p_{i,0}$ changes.

We define NST agents as agents whose pdf does not change as the number of actions m increases. Hence, let m_{i0} be the minimum number of actions necessary to define the pdf of an NST agent i. We have that $\forall m, m' \geq m_{i0}$, $\forall j \leq m_{i0}$ $p_{i,j}(m) = p_{i,j}(m')$, $\forall j > m_{i0}$ $p_{i,j}(m) = 0$.

We first give an intuitive description of the concept of diversity, then define formally diverse teams. By diversity, we mean agents that tend to disagree. In [8], a diverse team is defined as a set of agents with different pdfs. Hence, they disagree because of having different probabilities of playing certain actions. In this paper, we generalize their definition to capture cases where agents disagree on actions, regardless of whether their pdfs are the same or not. Formally, we define a diverse team to be one consisting of a set of ST agents (either different ST agents or copies of the same ST agent). In our theoretical development we will show that this definition captures the notion of diversity: a team of ST agents will tend to not agree on the same suboptimal actions. We call uniform team as the team composed by copies of an NST agent. This is an idealization to perform our initial analysis. We will later discuss more complex domains, where the agents of the uniform team also behave like ST agents.

3.1 A Hard Problem to a Diverse Team

We start with an example, to give an intuition about our model. Consider the agents in Table 1(a), where we show the pdf of the agents, and p_{best} of the uniform team (three copies of agent 1) and the diverse team (one copy of each agent). We assume agent 1 is an NST agent, while agent 2 and 3 are ST agents. In this situation the uniform team plays better than the diverse team. Now let's add one more action to the problem. Because agent 2 and 3 are ST agents, the probability mass on action 2 scatters to the newly added action (Table 1(b)). Hence, while before the ST agents would always agree on the same suboptimal action if they both did not vote for the optimal action, now they might vote for different suboptimal actions, creating a tie between each suboptimal action and the optimal one. Because ties are broken randomly, when this happens there will be a 1/3 chance that the tie will be broken in favor of the optimal action. Hence, p_{best} increases when the probability of the ST agents agreeing on the same suboptimal actions decreases, and the diverse team now plays better than the uniform team, even though individually agents 2 and 3 are weaker than agent 1.

Agents	Action 1	Action 2
Agent 1	0.6	0.4
Agent 2	0.55	0.45
Agent 3	0.55	0.45
Uniform p_{best}:	0.648	
Diverse p_{best}:	0.599	

(a) With 2 actions, the uniform team plays better than the diverse team

Agents	Action 1	Action 2	Action 3
Agent 1	0.6	0.4	0
Agent 2	0.55	0.25	0.2
Agent 3	0.55	0.15	0.3
Uniform p_{best}:	0.648		
Diverse p_{best}:	0.657		

(b) When we add one more action, the diverse team plays better than the uniform team

Table 1: The performance of a diverse team increases when we increase the number of available actions.

We now present our theoretical work. First we show that the performance of a diverse team converges when $m \to \infty$, to a value that is higher than the performance for any other m.

Theorem 1. *$p_{best}(m)$ of a diverse team of n agents converges to a certain value $\tilde{p}_{best}$ as $m \to \infty$. Furthermore, $\tilde{p}_{best} \geq p_{best}(m)$, $\forall m$.*

Proof. Let $p_{i,min} = \min_{j \in \mathbf{D_m}} p_{i,j}$, $p_{i,max} = \max_{j \in \mathbf{D_m}} p_{i,j}$ and $\mathbf{T}$ be the set of agents in the team. By our assumptions, there is a constant α such that $p_{i,max} \leq \alpha p_{i,min}$ for all agents i. Then, we have that $1 \geq 1 - p_{i,0} = \sum_{j \in \mathbf{D_m}} p_{i,j} \geq d_m p_{i,min}$. Therefore, $p_{i,min} \leq \frac{1}{d_m} \to 0$ as d_m tends to ∞ with m. Similarly, $\alpha p_{i,min} \to 0$ as $d_m \to \infty$. As $p_{i,j} \leq \alpha p_{i,min}$ we have that $\forall j$ $p_{i,j} \to 0$ as $d_m \to \infty$. We show that this implies that when $m \to \infty$, weak agents never agree on the same suboptimal action. Let i_1 and i_2 be two arbitrary agents. Without loss of generality, assume i_2's d_m ($d_m^{(i_2)}$) is greater than or equal i_1's d_m ($d_m^{(i_1)}$). The probability (σ_{i_1,i_2}) of i_1 and i_2 agreeing on the same suboptimal action is upper bounded by $\sigma_{i_1,i_2} = \sum_{a_j \in \mathbf{A} \setminus a_0} p_{i_1,j} p_{i_2,j} \leq d_m^{(i_2)} p_{i_1,max} p_{i_2,max} \leq d_m^{(i_2)} \alpha p_{i_2,min} p_{i_1,max} \leq \alpha p_{i_1,max}$ (as $d_m^{(i_2)} p_{i_2,min} \leq 1$). We have that $\alpha p_{i_1,max} \to 0$ as $p_{i_1,max} \to 0$, because α is a constant. Hence the probability of any two agents agreeing on a suboptimal action is $\frac{\sum_{i_1 \in \mathbf{T}} \sum_{i_2 \in \mathbf{T}, i_2 \neq i_1} \sigma_{i_1,i_2}}{2} \leq \frac{n(n-1)}{2} \max_{i_1,i_2} \sigma_{i_1,i_2} \to 0$, as n is a constant.

Hence, when $m \to \infty$, the diverse team only chooses a suboptimal action if all agents vote for a different suboptimal action or in a tie between the optimal action and suboptimal actions (because ties are broken randomly). Therefore, p_{best} converges to:

$$\tilde{p}_{best} = 1 - \prod_{i=1}^{n}(1 - p_{i,0}) - \sum_{i=1}^{n}(p_{i,0}\prod_{j=1,j\neq i}^{n}(1 - p_{j,0}))\frac{n-1}{n}, \quad (1)$$

that is, the total probability minus the cases where the best action is not chosen: the second term covers the case where all agents vote for a suboptimal action and the third term covers the case where one agent votes for the optimal action and all other agents vote for suboptimal actions.

When m is finite, the agents might choose a suboptimal action by agreeing over that suboptimal action. Therefore, we have that $p_{best}(m) \leq \tilde{p}_{best}$ $\forall m$. □

Let $p_{best}^{uniform}(m)$ be p_{best} of the uniform team, with m actions. A uniform team is not affected by increasing m, as the pdf of an *NST* agent will not change. Hence, $p_{best}^{uniform}(m)$ is the same, $\forall m$. If $\tilde{p}_{best}$ is high enough so that $\tilde{p}_{best} \geq p_{best}^{uniform}(m)$, the diverse team will overcome the uniform team, when $m \to \infty$. Therefore, the diverse team will be better than the uniform team when m is large enough.

In practice, a uniform team made of copies of the best agent might not behave exactly like a team of *NST* agents, as the best agent could also increase its d_m as m gets larger. We discuss this situation in Section 4. In order to perform that study, we derive in the following corollary how fast p_{best} converges to $\tilde{p}_{best}$, as a function of d_m.

Corollary 1. *$p_{best}(m)$ of a diverse team increases to $\tilde{p}_{best}$ in the order of $O(\frac{1}{d_m^{min}})$ and $\Omega(\frac{1}{d_m^{max}})$, where d_m^{max} is the highest and d_m^{min} the lowest d_m of the team.*

Proof. We assume here the notation that was used in the previous proof. First we show a lowerbound on $p_{best}(m)$. We have that $p_{best}(m) = 1 - \psi_1$, where ψ_1 is the probability of the team picking a suboptimal action. $\psi_1 = \psi_2 + \psi_3$, where ψ_2 is the probability of no agent agreeing and the team picks a suboptimal action and ψ_3 is the probability of at least two agents agreeing and the team picks a suboptimal action. Hence, $p_{best}(m) = 1 - \psi_2 - \psi_3 = \tilde{p}_{best} - \psi_3 \geq \tilde{p}_{best} - \psi_4$, where ψ_4 is the probability of at least two agents agreeing. Let $\sigma^{max} = \max_{i_1,i_2}\sigma_{i_1,i_2}$, and i_1^* and i_2^* are the agents whose $\sigma_{i_1^*,i_2^*} = \sigma^{max}$. We have that $p_{best}(m) \geq \tilde{p}_{best} - \frac{n(n-1)}{2}\sigma^{max} \geq \tilde{p}_{best} - \frac{n(n-1)}{2}d_m^{(i_2^*)}p_{i_1^*,max}p_{i_2^*,max} \geq \tilde{p}_{best} - \frac{n(n-1)}{2}d_m^{(i_2^*)}\alpha p_{i_1^*,min}\alpha p_{i_2^*,min} \geq \tilde{p}_{best} - \frac{n(n-1)}{2}\alpha^2\frac{1}{d_m^{(i_1^*)}}$ (as $p_{i,min} \leq \frac{1}{d_m}$). Hence, $p_{best}(m) \geq \tilde{p}_{best} - \frac{n(n-1)}{2}\alpha^2\frac{1}{d_m^{min}} \rightsquigarrow \tilde{p}_{best} - p_{best}(m) \leq O(\frac{1}{d_m^{min}})$.

Now we show an upper bound: $p_{best}(m) = \tilde{p}_{best} - \psi_3 \leq \tilde{p}_{best} - \psi_5$, where ψ_5 is the probability of at least two agents agreeing and no agents vote for the optimal action. Let $\sigma^{min} = \min_{i_1,i_2}\sigma_{i_1,i_2}$; i_1^* and i_2^* are the agents whose $\sigma_{i_1^*,i_2^*} = \sigma^{min}$; and $p_{max,0} = \max_{i\in\mathbf{T}} p_{i,0}$. Without loss of generality, we assume that $d_m^{(i_2^*)} \geq d_m^{(i_1^*)}$. Hence, $p_{best}(m) \leq \tilde{p}_{best} - \frac{n(n-1)}{2}\sigma^{min}(1-p_{max,0})^{n-2} \leq \tilde{p}_{best} -$

$\frac{n(n-1)}{2} d_m^{(i_1^*)} p_{i_1^*,min} p_{i_2^*,min} (1 - p_{max,0})^{n-2} \leq \tilde{p}_{best} - \frac{n(n-1)}{2} d_m^{(i_1^*)} \frac{p_{i_1^*,max} p_{i_2^*,max}}{\alpha^2} (1 - p_{max,0})^{n-2} \leq \tilde{p}_{best} - \frac{n(n-1)}{2} \alpha^{-2} \frac{1}{d_m^{i_2^*}} (1 - p_{max,0})^{n-2} \leq \tilde{p}_{best} - \frac{n(n-1)}{2} \alpha^{-2} \frac{1}{d_m^{max}} (1 - p_{max,0})^{n-2} \rightsquigarrow \tilde{p}_{best} - p_{best}(m) \geq \Omega(\frac{1}{d_m^{max}})$. $\square$

Hence, agents that change their d_m faster will converge faster to $\tilde{p}_{best}$. This is an important result when we consider later more complex scenarios where the d_m of the agents of the uniform team also change.

Note that $\tilde{p}_{best}$ depends on the number of agents n (Equation 1). Now we show that the diverse team tends to always play the optimal action, as $n \to \infty$.

Theorem 2. *$\tilde{p}_{best}$ converges to 1, as $n \to \infty$. Furthermore, $1 - \tilde{p}_{best}$ converges exponentially to 0, that is, $\exists$ constant c, such that $1 - \tilde{p}_{best} \leq c(1 - \frac{\epsilon}{2})^n$, $\forall n \geq \frac{2}{\epsilon}$. However, the performance of the uniform team improves as $n \to \infty$ only if $p_{s,0} = \max_j p_{s,j}$, where s is the best agent.*

Proof. By the previous proof, we know that when $m \to \infty$ the diverse team plays the optimal action with probability given by $\tilde{p}_{best}$. We show that $1 - \tilde{p}_{best} \to 0$ exponentially as $n \to \infty$ (this naturally induces $\tilde{p}_{best} \to 1$). We first compute an upper bound for $\sum_{i=1}^{n} (p_{i,0} \prod_{j=1, j\neq i}^{n} (1 - p_{j,0}))$: $\sum_{i=1}^{n} p_{i,0} \prod_{j=1, j\neq i}^{n} (1 - p_{j,0}) \leq \sum_{i=1}^{n} p_{i,0} (1 - p_{min,0})^{n-1} \leq n p_{max,0} (1 - p_{min,0})^{n-1} \leq n(1-\epsilon)^{n-1}$ for $p_{max,0} = max_i p_{i,0}$, $p_{min,0} = min_j p_{j,0}$.

Since $\prod_{i=1}^{n} (1 - p_{i,0})) \leq (1 - \epsilon)^n$, thus we have that $1 - \tilde{p}_{best} \leq (1-\epsilon)^n + n(1-\epsilon)^{n-1}$. So we only need to prove that there exists a constant c such that $(1-\epsilon)^n + n(1-\epsilon)^{n-1} \leq c(1 - \frac{\epsilon}{2})^n$, as follows: $\frac{(1-\epsilon)^{n+1} + (n+1)(1-\epsilon)^n}{(1-\epsilon)^n + n(1-\epsilon)^{n-1}} = (1 - \epsilon) \frac{1-\epsilon+n+1}{1-\epsilon+n} = 1 - \epsilon + \frac{1-\epsilon}{1-\epsilon+n} \leq 1 - \frac{1}{2}\epsilon$, if $n \geq \frac{2}{\epsilon}$ (by setting $\frac{1-\epsilon}{1-\epsilon+n} \leq \frac{\epsilon}{2}$). Hence, $\exists c$, such that $(1-\epsilon)^n + n(1-\epsilon)^{n-1} \leq c(1 - \frac{\epsilon}{2})^n$ when $n \geq \frac{2}{\epsilon}$. Therefore, the performance converges exponentially.

For the uniform team, the probability of playing the action that has the highest probability in the pdf of the best agent converges to 1 as $n \to \infty$ [11]. Therefore, the performance only increases as $n \to \infty$ if the optimal action is the one that has the highest probability. $\square$

Now we show that we can achieve further improvement in a diverse team by breaking ties in favor of the strongest agent.

Theorem 3. *When $m \to \infty$, breaking ties in favor of the strongest agent is the optimal tie-breaking rule for a diverse team.*

Proof. Let s be one of the agents. If we break ties in favor of s, the probability of voting for the optimal choice will be given by:

$$\tilde{p}_{best} = 1 - \prod_{i=1}^{n} (1 - p_{i0}) - (1 - p_{s0}) (\sum_{i=1, i\neq s}^{n} p_{i0} \prod_{j=1, j\neq i, j\neq s}^{n} (1 - p_{j0})) \qquad (2)$$

It is clear that Equation 2 is maximized by choosing agent s with the highest p_{s0}. However, we still have to show that it is better to break ties in favor of the strongest agent than breaking ties randomly. That is, we have to show that Equation 2 is always higher than Equation 1.

Equation 2 differs from Equation 1 only on the last term. Therefore, we have to show that the last term of Equation 2 is smaller than the last term of Equation 1. Let's begin by rewriting the last term of Equation 1 as:
$\frac{n-1}{n}\sum_{i=1}^{n} p_{i0}\prod_{j=1,j\neq i}^{n}(1-p_{j0}) = \frac{n-1}{n}(1-p_{s0})\sum_{i=1,i\neq s}^{n} p_{i0}\prod_{j=1,j\neq i,j\neq s}^{n}(1-p_{j0}) + \frac{n-1}{n}p_{s0}\prod_{j=1,j\neq s}^{n}(1-p_{j0})$

This implies that:
$\frac{n-1}{n}\sum_{i=1}^{n} p_{i0}\prod_{j=1,j\neq i}^{n}(1-p_{j0}) \geq \frac{n-1}{n}(1-p_{s0})\sum_{i=1,i\neq s}^{n} p_{i0}\prod_{j=1,j\neq i,j\neq s}^{n}(1-p_{j0})$.

We know that:
$(1-p_{s0})\sum_{i=1,i\neq s}^{n} p_{i0}\prod_{j=1,j\neq i,j\neq s}^{n}(1-p_{j0}) = \frac{n-1}{n}(1-p_{s0})\sum_{i=1,i\neq s}^{n} p_{i0}\prod_{j=1,j\neq i,j\neq s}^{n}(1-p_{j0}) + \frac{1}{n}(1-p_{s0})\sum_{i=1,i\neq s}^{n} p_{i0}\prod_{j=1,j\neq i,j\neq s}^{n}(1-p_{j0})$

Therefore, for the last term of Equation 2 to be smaller than the last term of Equation 1 we have to show that:
$\frac{n-1}{n}p_{s0}\prod_{j=1,j\neq s}^{n}(1-p_{j0}) \geq \frac{1}{n}(1-p_{s0})\sum_{i=1,i\neq s}^{n} p_{i0}\prod_{j=1,j\neq s,j\neq i}^{n}(1-p_{j0})$

It follows that this equation will be true if:
$p_{s0} \geq (1-p_{s0})\frac{\sum_{i=1,i\neq s}^{n} p_{i0}\prod_{j=1,j\neq i,j\neq s}^{n}(1-p_{j0})}{(n-1)\prod_{j=1,j\neq s}^{n}(1-p_{j0})} \rightsquigarrow p_{s0} \geq (1-p_{s0})\frac{1}{n-1}\sum_{i=1,i\neq s}^{n}\frac{p_{i0}}{(1-p_{i0})} \rightsquigarrow \frac{p_{s0}}{(1-p_{s0})} \geq \frac{\sum_{i=1,i\neq s}^{n}\frac{p_{i0}}{(1-p_{i0})}}{n-1}$

As s is the strongest agent the previous inequality is always be true. This is because $\frac{p_{s0}}{1-p_{s0}} = \frac{\sum_{i=1,i\neq s}^{n}\frac{p_{s0}}{(1-p_{s0})}}{n-1}$ and $\frac{p_{s0}}{1-p_{s0}} \geq \frac{p_{i0}}{(1-p_{i0})}\forall i \neq s$. Therefore, it is always better to break ties in favor of the strongest agent than breaking ties randomly. □

Next we show that with one additional assumption, not only the diverse team converges to $\tilde{p}_{best}$, but also p_{best} monotonically increases with m. Our additional assumption is that higher utility actions have higher probabilities, i.e., if $U(a_j) \geq U(a_{j'})$, then $p_{i,j} \geq p_{i,j'}$.

Theorem 4. *The performance of a diverse team monotonically increases with m, if $U(a_j) \geq U(a_{j'})$ implies that $p_{i,j} \geq p_{i,j'}$.*

Proof. Let an event be the resulted choice set of actions of these n agents. We denote by $P(V)$ the probability of occurrence of any event in V (hence, $P(V) = \sum_{v\in V} p(v)$). We call it a winning event if in the event the action chosen by plurality is action 0 (including ties). We assume that for all agents i, if $U(a_j) \geq U(a_{j'})$, then $p_{i,j} \geq p_{i,j'}$.

We show by mathematical induction that we can divide the probability of multiple suboptimal actions into a new action and $p_{best}(m+1) \geq p_{best}(m)$. Let λ be the number of actions whose probability is being divided. The base case holds trivially when $\lambda = 0$. That is, there is a new action, but all agents have a 0 probability of voting for that new action. In this case we have that p_{best} does not change, therefore $p_{best}(m+1) \geq p_{best}(m)$.

Now assume that we divided the probability of λ actions and it is true that $p_{best}(m+1) \geq p_{best}(m)$. We show that it is also true for $\lambda+1$. Hence, let's pick one more action to divide the probability. Without loss of generality, assume it is action a_{d_m}, for agent c, and its probability is being divided into action a_{d_m+1}. Therefore, $p'_{c,d_m} = p_{c,d_m} - \beta$ and $p'_{c,d_m+1} = p_{c,d_m+1} + \beta$, for $0 \leq \beta \leq p_{c,d_m}$. Let $p_{best}^{after}(m+1)$ be the probability of voting for the best action after this new division, and $p_{best}^{before}(m+1)$ the probability before this new division. We show that $p_{best}^{after}(m+1) \geq p_{best}^{before}(m+1)$.

Let Γ be the set of all events where all agents voted, except for agent c (the order does not matter, so we can consider agent c is the last one to post its vote). If $\gamma \in \Gamma$ will be a winning event no matter if agent c votes for a_{d_m} or a_{d_m+1}, then changing agent c's pdf will not affect the probability of these winning events. Hence, let $\Gamma' \subset \Gamma$ be the set of all events that will become a winning event depending if agent c does not vote for a_{d_m} or a_{d_m+1}. Given that $\gamma \in \Gamma'$ already happened, the probability of winning or losing is equal to the probability of agent c not voting for a_{d_m} or a_{d_m+1}.

Now let's divide Γ' in two exclusive subsets: $\Gamma_{d_m+1} \subset \Gamma'$, where for each $\gamma \in \Gamma_{d_m+1}$ action a_{d_m+1} is in tie with action a_0, so if agent c does not vote for a_{d_m+1}, γ will be a winning event; $\Gamma_{d_m} \subset \Gamma'$, where for each $\gamma \in \Gamma_{d_m}$ action a_{d_m} is in tie with action a_0, so if agent c does not votes for a_{d_m}, γ will be a winning event. We do not consider events where both a_{d_m+1} and a_{d_m} are in tie with a_0, as in that case the probability of a winning event does not change (it is given by $1 - p'_{c,d_m} - p'_{c,d_m+1} = 1 - p_{c,d_m} - p_{c,d_m+1}$).

Note that for each $\gamma \in \Gamma_{d_m+1}$, the probability of a winning event equals $1 - p'_{c,d_m+1}$. Therefore, after changing the pdf of agent c, for each $\gamma \in \Gamma_{d_m+1}$, the probability of a wining event decreases by β. Similarly, for each $\gamma \in \Gamma_{d_m}$, the probability of a winning event equals $1 - p'_{c,d_m}$. Therefore, after changing the pdf of agent c, for each $\gamma \in \Gamma_{d_m}$, the probability of a winning event increases by β.

Therefore, $p_{best}^{after}(m+1) \geq p_{best}^{before}(m+1)$ if and only if $P(\Gamma_{d_m}) \geq P(\Gamma_{d_m+1})$. Note that $\forall \gamma \in \Gamma_{d_m+1}$ there are more agents that voted for a_{d_m+1} than for a_{d_m}. Also, $\forall \gamma \in \Gamma_{d_m}$ there are more agents that voted for a_{d_m} than for a_{d_m+1}. If, for all agents i, $p_{i,d_m} \geq p_{i,d_m+1}$, we have that $P(\Gamma_{d_m}) \geq P(\Gamma_{d_m+1})$. Therefore, $p_{best}^{after}(m+1) \geq p_{best}^{before}(m+1)$, so we still have that $p_{best}(m+1) \geq p_{best}(m)$. Also note that for the next step of the induction be valid, so that we can still divide the probability of one more action, it is necessary that $p'_{c,d_m} \geq p'_{c,d_m+1}$. $\square$

3.2 Generalizations

In the previous theorems we focused on the probability of playing the best action, assuming that $U(a_0) \gg U(a_j) \forall j \neq 0$. We show now that the theorems still hold in more general domains where r actions ($\mathbf{A_r} \subset \mathbf{A}$) have a significant high utility, i.e., $U(a_{j_1}) \gg U(a_{j_2}) \forall j_1 < r, j_2 \geq r$. Hence, we now focus on the probability of playing any action in $\mathbf{A_r}$. We assume that our assumptions are also generalized, i.e., $p_{i,j} > \epsilon \forall j < r$, and the number d_m of suboptimal actions (a_j, $j \geq r$) in the $\mathbf{D_m}$ set increases with m for ST agents.

Theorem 5. *The previous theorems generalize to settings where* $U(a_{j_1}) \gg U(a_{j_2})$ $\forall j_1 < r, j_2 \geq r$.

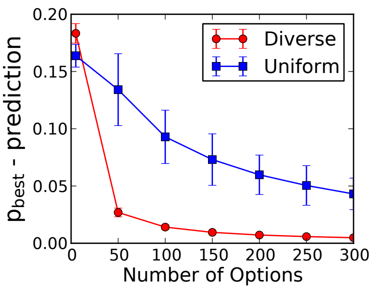
Proof Sketch We give here a proof sketch. We just have to generate new pdfs $p'_{i,j}$, such that $p'_{i0} = \sum_{j=0}^{r-1} p_{i,j}$, and $p'_{i,b} = p_{i,b+r-1}, \forall b \neq 0$. We can then reapply the proofs of the previous theorems, but replacing $p_{i,j}$ by $p'_{i,j}$. Note that this does not guarantee that all agents will tend to agree on the same action in $\mathbf{A_r}$; but the team will still tend to pick any action in $\mathbf{A_r}$, since the agents are more likely to agree on actions in $\mathbf{A_r}$ than on actions in $\mathbf{A} \setminus \mathbf{A_r}$. □

Now we discuss a different generalization: what happens when $p_{i,0}$ decreases as m increases ($\forall$ agents i). If $p_{i,0} \to \tilde{p}_{i,0}$ as $m \to \infty$, the performance in the limit for a diverse team will be $\tilde{p}_{best}$ evaluated at $\tilde{p}_{i,0}$. Moreover, even if $p_{i,0} \to 0$, our conclusions about relative team performance are not affected as long as we are comparing two ST teams that have similar $p_{i,0}$: the same argument as in Corollary 1 implies that the team with faster growing d_m will perform better.

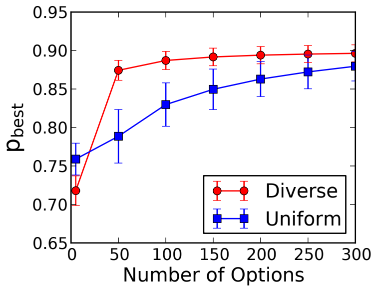
4 Experimental Analysis

4.1 Synthetic Experiments

We present synthetic experiments, in order to better understand what happens in real systems. We generate agents by randomly creating pdfs and calculate the probability of playing the best action (p_{best}) of the generated teams. The details of these experiments are available in the online appendix (in `http://teamcore.usc.edu/people/sorianom/coin2014-appendix.pdf`).



(a) Convergence of p_{best} to predicted value.



(b) Actual performance value.

Fig. 1: Comparing diverse and uniform when uniform also increases d_m.

As we said earlier, uniform teams composed by NST agents is an idealization. In more complex domains, the best agent will not behave exactly like an NST

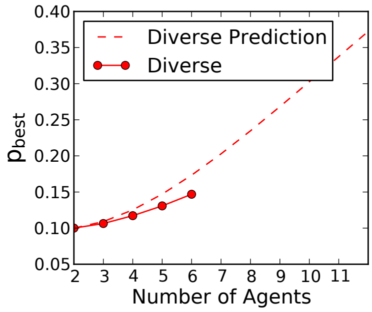


Fig. 2: p_{best} of a diverse team as the number of agents increases.

agent, its d_m will also increase. We perform synthetic experiments to study this situation. We consider that the best agent is still closer to an *NST* agent, therefore it increases its d_m at a slower rate than the agents of the diverse team. We can see the average result for 200 random teams in Figure 1, where in Figure 1(a) we show the difference between the performance in the limit ($\tilde{p}_{best}$) and the actual $p_{best}(m)$ for the diverse and the uniform teams; in Figure 1(b) we show the average $p_{best}(m)$ of the teams. As can be seen, when the best agents increase their d_m at a slower rate than the agents of the diverse team, the uniform teams converge slower to $\tilde{p}_{best}$. Even though they play better than the diverse teams for a small m, they are surpassed by the diverse teams as m increases. However, because $\tilde{p}_{best}$ of the uniform teams is actually higher than the one of the diverse teams, eventually the performance of the uniform teams get closer to the performance of the diverse teams, and will be better than the one of the diverse teams again for a large enough m.

This situation is expected according to Theorem 1. If the d_m of the best agent also increases as m gets larger, the uniform team will actually behave like a diverse team and also converge to $\tilde{p}_{best}$. $\tilde{p}_{best}^{uniform} \geq \tilde{p}_{best}^{diverse}$, as the best agent has a higher probability of playing the optimal action. Hence, in the limit the uniform team will play better than the diverse team. However, as we saw in Corollary 1, the speed of convergence is in the order of $1/d_m$. Therefore, the diverse team will converge faster, and can overcome the uniform team for moderately large m.

As Theorem 2 only holds when $m \to \infty$, we also explore the effect of increasing the number of agents for a large m. The $\tilde{p}_{best}$ of a team of agents is shown as the dashed line in Figure 2. We are plotting for agents that have a probability of playing the best action of only 10%, but as we can see the probability quickly grows as the number of agents increases. We also calculate p_{best} for random teams from 2 to 6 agents (shown as the continuous line), when there are 300 available actions. As can be seen, the teams have a close performance to the expected. We only show up to 6 agents because it is too computationally expensive to calculate the pdfs of larger teams.

4.2 Computer Go

We present now results in a real system. We use in our experiments 4 different Go software: Fuego 1.1, GnuGo 3.8, Pachi 9.01, MoGo 4, and two (weaker) variants of Fuego (FuegoΔ and FuegoΘ), in a total of 6 different, publicly available, agents. Fuego is considered the strongest agent among all of them. Fuego is an implementation of the UCT Monte Carlo Go algorithm, therefore it uses heuristics to simulate games in order to evaluate board configurations. Fuego uses mainly 5 heuristics during these simulations, and they are executed in a hierarchical order. The original Fuego agent follows the order <Atari Capture, Atari Defend, Lowlib, Pattern> (The heuristic called Nakade is not enabled by default). Our variation called FuegoΔ follows the order <Atari Defend, Atari Capture, Pattern, Nakade, Lowlib>, while FuegoΘ follows the order <Atari Defend, Nakade, Pattern, Atari Capture, Lowlib>. Also, FuegoΔ and FuegoΘ have half of the memory available when compared with the original Fuego.

All our results are obtained by playing either 1000 games (to evaluate individual agents) or 2000 games (to evaluate teams), in a HP dl165 with dual dodeca core, 2.33GHz processors and 48GB of RAM. We compare results obtained by playing against a fixed opponent. Therefore, we evaluate systems playing as white, against the original Fuego playing as black. We removed all databases and specific board size knowledge of the agents, including the opponent. We call Diverse as the team composed of all 6 agents, and Uniform as the team composed of 6 copies of Fuego. Each agent is initialized with a different random seed, therefore they will not vote for the same action all the time in a given world state, due to the characteristics of the search algorithms. In the graphs we show in this section, the error bars show the confidence interval (99% of confidence).

We evaluate the performance of the teams over 7 different board sizes. We changed the time settings of individual agents as we increased the board size, in order to keep their strength as constant as possible. The average winning rates of the team members is shown in Table 2, and in the appendix we show the winning rates of the individual agents. [3]

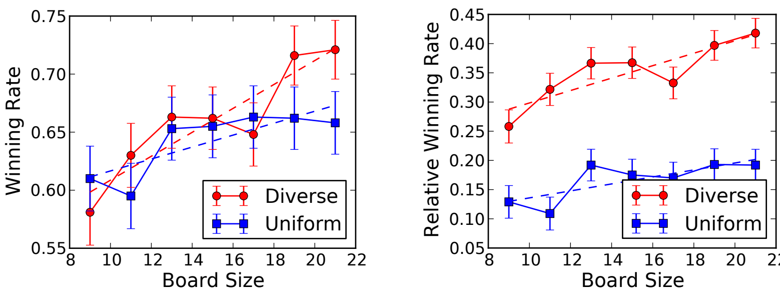
Team	9x9	11x11	13x13	15x15	17x17	19x19	21x21
Diverse	32.2%	30.8%	29.6%	29.4%	31.5%	31.9%	30.3%
Uniform	48.1%	48.6%	46.1%	48.0%	49.3%	46.9%	46.6%

Table 2: Average winning rates of the team members across different board sizes.

We can see our results in Figure 3 (a). Diverse improves from 58.1% on 9x9 to 72.1% on 21x21, an increase in winning rate that is statistically significant with

[3] In our first experiment, Diverse improved from 56.1% on 9x9 to 85.9% on 19x19. We noted, however, that some of the diverse agents were getting stronger in relation to the opponent as the board size increased. Hence, by changing the time setting to keep the strength constant, we are actually making our claims harder to show, not easier.

$p < 2.2 \times 10^{-16}$. This result is expected according to Theorem 1. The Uniform team changes from 61.0% to 65.8%, a statistically significant improvement with $p = 0.0018$. As we saw before, an increase in the performance of Uniform can also be expected, as the best agent might not be a perfect NST agent. A linear regression of the results of both teams gives a slope of 0.010 for the diverse team (adjusted R^2: 0.808, $p = 0.0036$) and 0.005 for the uniform team (adjusted R^2: 0.5695, $p = 0.0305$). Therefore, the diverse team improves its winning rate faster than the uniform team. To check if this is a significant difference, we evaluate the interaction term in a linear regression with multiple variables. We find that the influence of board size is higher on Diverse than on Uniform with $p = 0.0797$ (estimated coefficient of "size of the board * group type": -10.321, adjusted R^2: 0.7437). Moreover, on the 9x9 board Diverse is worse than Uniform ($p = 0.0663$), while on the 21x21 board Diverse is better with high statistical significance ($p = 1.941 \times 10^{-5}$). We also analyze the performance of the teams subtracted by the average strength of their members (Figure 3 (b)), in order to calculate the increase in winning rate achieved by "teamwork" and compensate fluctuations on the winning rate of the agents as we change the board size. Again, the diverse team improves faster than the uniform team. A linear regression results in a slope of 0.0104 for Diverse (adjusted R^2: 0.5549, $p = 0.0546$) and 0.0043 for Uniform (adjusted R^2: 0.1283, $p = 0.258$).



(a) Absolute winning rates. (b) Relative to the average strength of team members.

Fig. 3: Winning rate in the real Computer Go system.

We also evaluate the performance of teams of 4 agents (Diverse 4 and Uniform 4). For Diverse 4, we removed FuegoΔ and FuegoΘ from the Diverse team. As can be seen in Figure 4, the impact of adding more agents is higher for the diverse team in a larger board size(21x21). In the 9x9 board, the difference between Diverse 4 and Diverse 6 is only 4.4%, while in 21x21, it is 14%. Moreover, we can see a higher impact of adding agents for the diverse team, than for the uniform team. These results would be expected according to Theorem 2.

As can be seen, the prediction of our theory holds: the diverse team improves significantly as we increase the action space. The improvement is enough to make

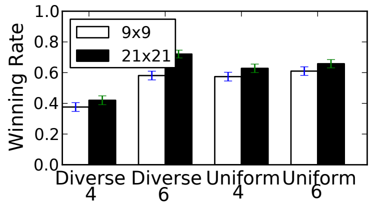


Fig. 4: Winning rates for 4 and 6 agents teams.

it change from playing worse than the uniform team on 9x9 to playing better than the uniform team with statistical significance on the 21x21 board. Furthermore, we show a higher impact of adding more agents when the size of the board is larger.

4.3 Analysis

To test the assumptions of our model, we estimate a pdf for each one of the agents. For each board size, and for each one of 1000 games from our experiments, we randomly choose a board state between the first and the last movement. We make Fuego evaluate the chosen board, but we give it a time limit 50x higher than the default one. Therefore, we use this much stronger version of Fuego to approximate the true ranking of all actions. For each board size, we run all agents in each board sample and check in which position of the approximated true ranking they play. This allow us to build a histogram for each agent and board size combination. Some examples can be seen in Figure 5. We can see that a strong agent, like Fuego, has most of its probability mass in the higher ranked actions, while weaker agents, like GnuGo, has the mass of its pdf distributed over a larger set of actions, creating a larger tail. Moreover, the probability mass of GnuGo is spread over a larger number of actions when we increase the size of the board.

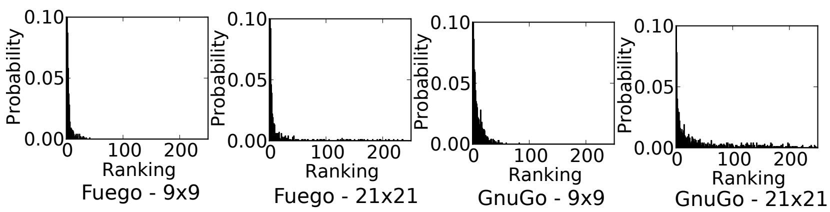
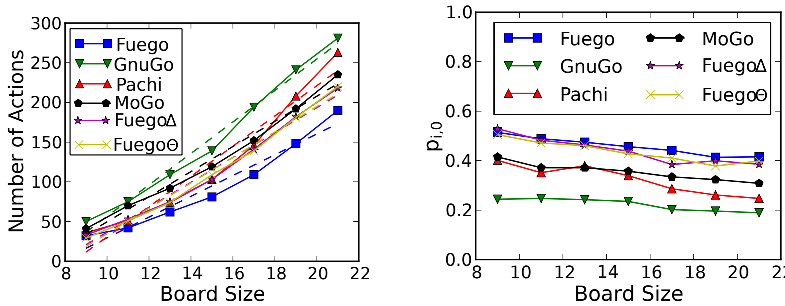


Fig. 5: Histograms of agents for different board sizes.

We study how the pdfs of the agents change as we increase the action space. Our hypothesis is that weaker agents will have a behavior closer to ST agents, while stronger agents to NST agents. In Figure 6(a) we show how many actions receive a probability higher than 0. As can be seen, Fuego does not behave exactly like an *NST* agent. However, it does have a slower growth rate than the other agents. A linear regression gives the following slopes: 13.08, 19.82, 19.05, 15.82, 15.69, 16.03 for Fuego, Gnugo, Pachi, Mogo, FuegoΔ and FuegoΘ, respectively (R^2: 0.95, 0.98, 0.94, 0.98, 0.98, 0.98, respectively). It is clear, therefore, that the probability mass of weak agents is distributed into bigger sets of actions as we increase the action space, and even though the strongest agent does not behave in the idealized way it does have a slower growth rate.

We also verify how the probability of playing the best action changes for each one of the agents as the number of actions increase. Figure 6(b) shows that even though all agents experience a decrease in p_{best}, it does not decrease much (on average, they decreased about 25% from 9x9 to 21x21).



(a) Size of the set of actions that receive a nonzero probability.

(b) p_{best} as the size of the board grows.

Fig. 6: Verifying the assumptions in the real system.

5 Conclusion

Diversity is an important point to consider when forming teams. In this paper we present a new model that captures better than previous ones the intuitive notion of diverse agents as agents that tend to disagree. This model allows us to make new predictions. We show that the performance of diverse teams increase as the size of the action space gets larger, and also that diverse teams converge faster than uniform teams. Besides, in large action spaces the performance of a diverse team converges exponentially fast to the optimal one as the number of agents increases. Experimental results with real Computer Go agents match the predictions of our theory. Hence, by showing theoretical and experimental results, this paper is a significant step towards better understanding how to form strong teams.

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